Received January 2002
Revised February 2003
Accepted April 2003

Aeroelastic analysis of turbomachinery

Part I – phase lagged boundary condition methods

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Keywords *Fourier analysis, Time series analysis, Multiple time series, Aircraft engines*

Abstract *In this two-part paper, aeroelastic analysis of turbomachinery blade rows and phase-lagged boundary conditions used for analysis are described. Part I of the paper describes a study of phase-lagged boundary condition methods used for non-zero interblade phase angle analysis. The merits of time-shifted (direct-store), Fourier decomposition and multiple passage methods are compared. These methods are implemented in a time marching Euler/Navier-Stokes solver and are applied to a fan for subsonic and supersonic inflow and to a turbine geometry with supersonic exit flow. Results showed good comparisons with published results and measured data. The time-shifted and Fourier decomposition methods compared favorably in computational costs with respect to multiple passage analysis despite a slower rate of convergence. The Fourier-decomposition method was found to be better suited for workstation environment as it required significantly less storage, although at the expense of slightly higher computational cost. The time-shifted method was found to be better suited for computers where fast input-output devices are available.*

Introduction

Aeroelastic problems contribute significantly to the development and maintenance costs of aircraft engines. In turbomachinery, both flutter and synchronous vibration can cause blade failures. Flutter problems, usually detected during the design phase, cause significant program delays and cost overruns. Numerical methods are being developed to help design flutter free turbomachinery components. This two-part paper details the study carried out to address the issues associated with the aeroelastic analysis of turbomachinery components. Part I of the paper investigates the advantages and disadvantages associated with methods used to carry out the non-zero

The authors are grateful to Dr Daniel Hoyniak for his significant help in analyzing the turbine configuration. This work was carried out under a grant from the NASA Advanced Subsonic Technology Project. The numerical results were obtained on the CRAY C-90 computer at the NAS research facility of NASA.

International Journal of Numerical Methods for Heat & Fluid Flow
Vol. 14 No. 3, 2004
pp. 366-381

0961-5539
DOI 10.1108/09615530410518002

interblade phase angle (IBPA) analysis of turbomachinery rotors. In Part II – stability computations, the details of an aeroelastic stability analysis are listed.

Computational costs associated with three-dimensional numerical aeroelastic analysis methods are prohibitive for even a moderate sized problem. Three-dimensional analyses capture all the required physics but are computationally very expensive, especially for cases with non-zero IBPAs, which require modeling of a large number of blade passages. Some aeroelastic problems require analysis of the full annulus of blade row (Breard *et al.*, 2000; Marshall *et al.*, 2000). However, large number of aeroelastic problems can make use of the symmetry of the problem and use phase-lagged methods to reduce computational cost. These methods reduce the computational domain to a single blade passage for any IBPA. Several phase-lagged boundary condition methods have been reported earlier. Erdos *et al.* (1977) developed an analysis based on the direct-store method. The direct-store method stores all the relevant fluid properties over the oscillation cycle which is applied with appropriate lag for the IBPA being analyzed. Although no loss in fidelity occurs, the method requires significant additional memory and may become prohibitive for large three-dimensional problems. Another method, known as time-inclined computational plane approach, was proposed by Giles (1988) primarily to overcome the problem encountered in rotor-stator applications where no final periodic state exists. This method requires transforming the original governing equation to account for the tilting of the time plane. He (1989) proposed a shape-correction method for applying the phase-lagged boundary conditions that did not have the storage penalty associated with the direct-store method. In the shape-correction method, the variation in fluid properties over an oscillation cycle is decomposed into its Fourier coefficients and only the coefficients are stored. These coefficients are used to regenerate the fluid properties as required. For most flow conditions only a few coefficients need to be stored and used for regeneration, significantly reducing the required storage. Later, He and Denton (1994) extended the method to three-dimensions. Peitsch *et al.* (1996) proposed a variation of the direct-store method to reduce the storage requirements, using a "foothold technique" that stores the fluid properties only at certain foothold points. The properties at other points over the oscillation cycle are obtained by interpolation from the nearest foothold points.

Except for the Giles' "time-tilting" method, which requires transforming the governing equations, the above-mentioned methods are based on either the direct-store method or the shape-correction method. The direct-store method requires large memory for storing the flowfield properties over the oscillation cycle, whereas the Fourier decomposition (FD) method requires additional computational time to Fourier decompose and then regenerate the fluid properties using stored coefficients. Also, if strong vibrating shocks are present at the periodic passage boundary, a few coefficients may not be able to accurately model the discontinuity. This may require significantly large number of coefficients, potentially offsetting the benefits of reduced storage requirements. Further, due to the lag associated with enforcing the "phase-lag", the rate of convergence also becomes an issue for the two methods as compared to multiple passage analysis method. Clearly, there are advantages and disadvantages associated with various methods. It is not clear if one method is superior to the other. The present study attempts to highlight the advantages of each of the methods, given the problem of interest and resources at hand.

For the present study, a three-dimensional multi-stage aeroelastic analysis code TURBO-AE (Bakhle *et al.*, 1997), based on Euler/Navier-Stokes equations is used. The two phase-lagged methods: time-shifted (TS) and FD, along with the multiple passage (MP) analysis are all implemented in the TURBO-AE. The study presented in Part I applies the two phase-lagged methods along with the MP method to a fan and a turbine configuration. The results obtained are compared for accuracy and efficiency of the methods. To reduce computational cost, results are obtained using inviscid calculations.

Non-zero IBPA analysis

The turbomachine blade rows can undergo instability in any one of the possible IBPAs for a given frequency and mode shape. The possible IBPAs depend on the number of blades in the rotor and are defined as (Janus, 1989)

$$\phi = \frac{2\pi j}{\mathrm{NB}} \quad j = 1,\ \mathrm{NB} - 1 \tag{1}$$

where ϕ is the IBPA and NB is the number of blades in the rotor. MP, TS, and FD methods are used in the present analysis to calculate the stability characteristics of non-zero IBPA motion.

MP method

Figure 1 shows a typical non-zero IBPA motion. For the IBPA of 120° shown in the figure, every fourth blade will undergo identical blade motion. With this

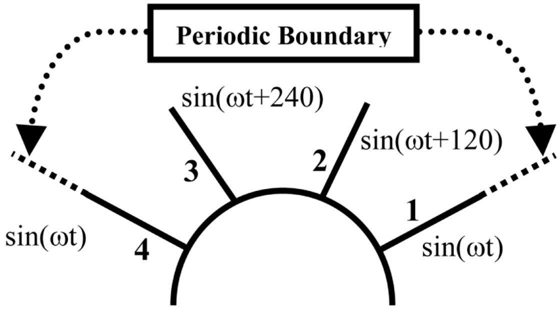


Figure 1.
Non-zero IBPA vibration

symmetry, the blade row can be analyzed using only three blade passages and enforcing periodicity across the fluid boundaries associated with blades 1 and 4.

For any given IBPA ϕ, if

$$\mathrm{mod}(2\pi, \phi) = 0 \quad (2)$$

then the required number of blade passages NPAS for the analysis can be determined as

$$\mathrm{NPAS} = \frac{2\pi}{\phi}, \quad \phi \neq 0 \quad (3)$$

otherwise, NPAS = NB, or all blade passages will need to be included in the analysis. For $\phi = 0$ (also $= 2\pi$) IBPA, one blade passage will be needed. Clearly, for several cases, all the blades in the blade row have to be included in the analysis. Further, since NPAS is inversely proportional to ϕ, smaller values of ϕ, will require larger number of blade passages for analysis. This will make the computational cost prohibitive for a large number of problems of interest. The phase-lagged boundary conditions help to reduce the computational cost by using a single blade passage for analyzing all IBPAs.

The phase-lagged methods are based on the assumption that if two adjacent blades oscillate with a phase difference, the fluid properties at the computational boundaries associated with these blades will also vary in time with the same phase, after the transients have died out. Thus, applying the fluid properties from one blade boundary with appropriate phase difference to the other boundary would result in reducing the computational domain to just one blade passage, significantly reducing the computational cost (Figure 2).

The phase-lagged boundary condition has been implemented in TURBO-AE using TS and FD methods.

TS method

Application of TS phase-lagged boundary condition requires first storing the time variation of the fluid properties at the blade passage boundaries. These values are then used to update the fluid properties associated with the other blade, appropriately shifted in time for the phase of blade motion (Figure 3).

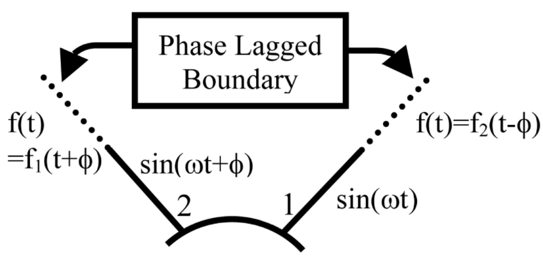


Figure 2.
Phase-lagged boundary

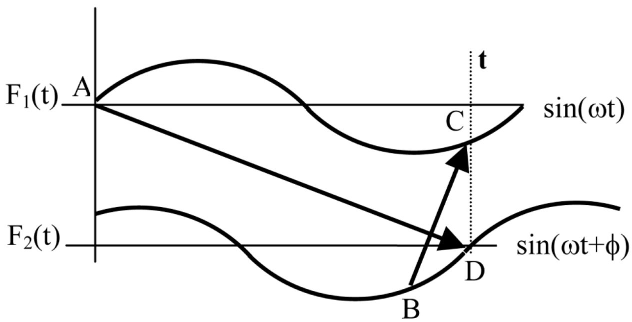


Figure 3.
TS analysis

For example, to simulate blade motion with ϕ IBPA, the fluid properties f_C and f_D at any time t (Figure 3) are obtained from the properties stored at time instants B and A, i.e. f_B and f_A, respectively. Equations (4) and (5) can be used to obtain the fluid properties from time instants A and B. The fluid properties at the interior boundary point of one boundary are stored and applied at the exterior boundary (ghost) point of the other boundary.

$$f_D = F_1\left[t - \left(T - \frac{\phi}{2\pi} * T\right)\right] = f_A \tag{4}$$

$$f_C = F_2\left[t - \frac{\phi}{2\pi} * T\right] = f_B \tag{5}$$

For parts of the initial blade vibration cycle, no prior information is available. During this part of the cycle, the boundaries are treated as being periodic. This introduces errors that get convected out of the computational domain in a few cycles of oscillation. The errors introduced in the initial cycle increase the number of oscillations required for convergence. Depending upon the size of the grid and the time steps used for simulating the oscillation cycle, prohibitively large in-core storage may be required. Storage requirements vary from a minimum of half a cycle for 180° IBPA to almost the entire cycle for the smallest IBPA possible for a given blade row. However, on machines with fast input-output (IO) devices, time histories of the fluid properties can be written on to a hard disk. This will significantly reduce the in-core storage, although at marginally higher computational cost.

FD method

Storage requirements of the TS method can be significantly reduced by using the FD method. Instead of storing the time history, as done in TS method, the time history is decomposed in its Fourier coefficients and only the coefficients are stored. In most cases it is sufficient to use only a few Fourier coefficients. The Fourier coefficients for any time dependent function $f(t)$ can be calculated as follows:

$$A_0 = \frac{1}{T}\sum_{j=1}^{\mathrm{NP}} f(t)\Delta t \tag{6}$$

$$A_n = \frac{\omega}{2\pi}\sum_{j=1}^{\mathrm{NP}} f(t)\sin(n\omega t)\Delta t \tag{7}$$

$$B_n = \frac{\omega}{2\pi}\sum_{j=1}^{\mathrm{NP}} f(t)\cos(n\omega t)\Delta t \tag{8}$$

where ω is the blade vibration frequency, n is the harmonic number, NP is the number of steps taken over one oscillation cycle, T is the time period and Δt is the time step size. These quantities are related as:

$$\omega T = 2\pi \tag{9}$$

$$\mathrm{NP} * \Delta t = T \tag{10}$$

The coefficients A_0, A_n, and B_n are stored for desired number of harmonics and are used to regenerate the fluid properties using the appropriate phase lag. For the example shown in Figure 3, the values at time t can be obtained using equations (11) and (12). The subscripts 1 and 2 denote the associated blades.

$$f_C = A_{02} + \sum_{n=1}^{\mathrm{N}} A_{n_2}\sin[n(\omega t - \phi)] + \sum_{n=1}^{\mathrm{N}} B_{n_2}\cos[n(\omega t - \phi)] \tag{11}$$

$$f_D = A_{01} + \sum_{n=1}^{\mathrm{N}} A_{n_1}\sin[n(\omega t + \phi)] + \sum_{n=1}^{\mathrm{N}} B_{n_1}\cos[n(\omega t + \phi)] \tag{12}$$

The Fourier coefficients are calculated from the fluid property variation over one time period. This implies that the coefficients are lagged by one cycle and no coefficient is available for the first cycle. Again, similar to the TS method, for the first vibration cycle the passage boundaries are treated as being periodic boundaries, adding to the delay in convergence. Further, unlike the TS method where the most current possible flow variable is used, one has to wait for one time period to update the Fourier coefficients. This further delays the convergence. The rate of convergence can be improved by updating the coefficients several times over one oscillation cycle by staggering the coefficient computation cycle (Figure 4). However, this increases not only the computational cost, as more coefficients need to be calculated, but also the storage requirements, as more number of coefficients need to be stored.

The turbo-AE code

The aeroelastic solver TURBO-AE is briefly described in this section. The solver can model multiple blade rows undergoing harmonic oscillations with arbitrary IBPAs. It is based on an Euler/Navier-Stokes unsteady aerodynamic solver for internal flow calculations of axial flow turbomachinery components TURBO (Lane, 1956). For viscous calculations, Reynolds-averaged Navier-Stokes equations are solved with Baldwin-Lomax turbulence model. The aerodynamic equations are solved using a finite volume scheme. Flux vector splitting is used to evaluate the flux Jacobians on the left-hand side. The right-hand side fluxes are discretized using the higher order total variation diminishing (TVD) scheme based on Roe's flux difference splitting. Newton sub-iterations are used at each time step to maintain the higher accuracy. Symmetric Gauss-Sidel iterations are applied to the discretized equations for improved convergence. A three-dimensional boundary condition based on eigen analysis (Montgomery and Verdon, 1997) is applied at the inlet/exit boundaries.

The aeroelastic characteristics of the rotor are obtained by calculating the energy exchange between the vibrating blade and its surrounding fluid. A positive work on the blade indicates instability. The aeroelastic analysis is carried out first by obtaining a steady aerodynamic solution for the given conditions. The blades are then forced into a prescribed harmonic motion (specified frequency and IBPA) and the unsteady aerodynamic behavior and work-per-cycle are calculated. The blade motion is simulated using a dynamic grid deformation technique. The grid is updated at each time step by recalculating the grid using linear interpolation, assuming the far field boundaries to be fixed. The grids on the casing are allowed to slide along the casing.

Numerical results

Results obtained from TURBO-AE for phase-lagged boundary conditions are presented in this section. Results obtained for a flat plate helical fan configuration used by Montgomery and Verdon (1997) are presented first. The fan configuration consists of 24 flat plate blades with zero thickness enclosed within a rigid cylindrical duct with no tip-gap. At mid-span, the stagger angle is 45° and the gap to chord ratio is one. Results are presented at mid-span based on two conditions:

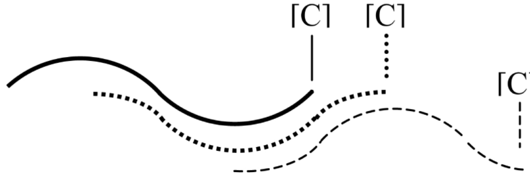


Figure 4.
Multiple updates of Fourier coefficients

(1) a subsonic relative inflow Mach number of 0.7 at zero incidence with axial Mach number of 0.495, and

(2) a supersonic relative inflow Mach number of 1.3 at zero incidence with axial Mach number of 0.9192.

An H-O grid with $141 \times 11 \times 41$ grid points is used.

A comparison of results for the subsonic flow condition from TURBO-AE, a linearized Euler analysis (Montgomery and Verdon, 1997), and linear theory (Smith, 1972) is shown in Figures 5 and 6 at the subsonic relative inflow condition. Figure 5 shows the comparisons for zero IBPA, and Figure 6 shows

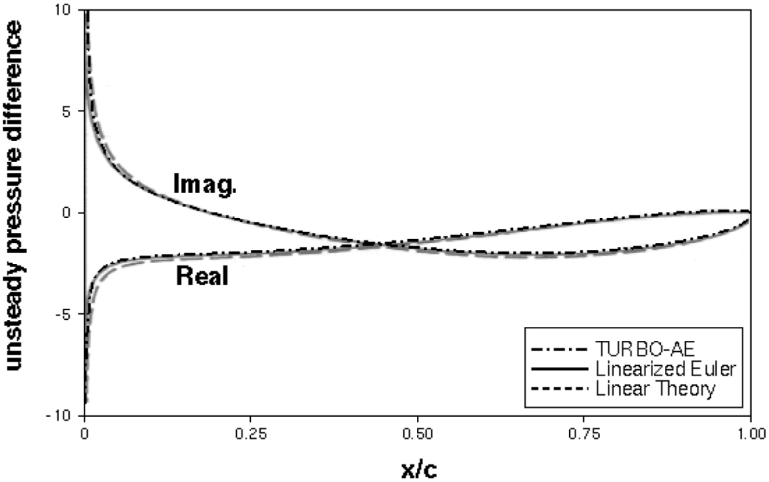


Figure 5. Unsteady pressure difference variation with chord at mid-span for 0° IBPA pitching oscillations

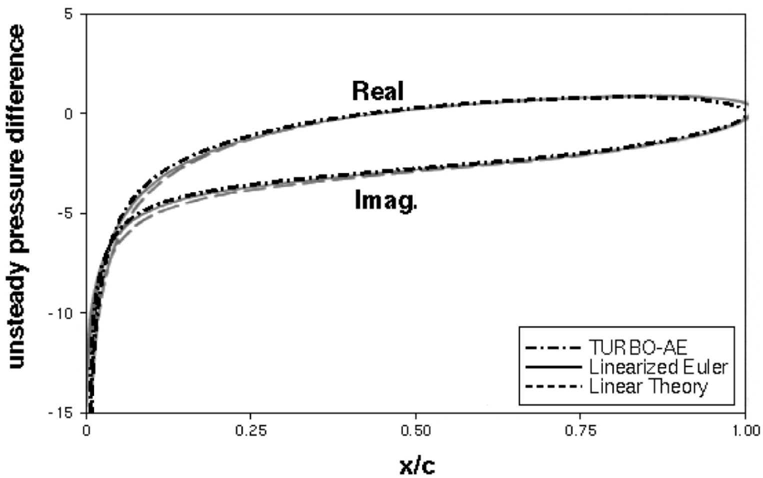


Figure 6. Unsteady pressure difference variation with chord at mid-span for 180° IBPA plunging oscillations

for 180 IBPA. For the 180 IBPA, the analysis was carried out using two blade passages to simulate the 180° phase motion. The unsteady pressure difference compares very well with results from the other two methods indicating that the unsteady aerodynamic pressures are accurately calculated by TURBO-AE.

The study of the phase-lagged analysis methods was carried out next for the subsonic relative inflow condition. The blades were forced into a pitching oscillation about their mid-chord at a reduced frequency of one and −90° IBPA. Two different analyses were performed for the FD method. In one of the analyses, one Fourier coefficient was retained and the coefficient was updated only once per oscillation cycle.

In the other analysis, the coefficient was updated four times during each oscillation cycle. The multiple updates of the coefficients is expected to provide a faster convergence. The results obtained from the FD method are compared with that obtained from the TS method (Figures 7 and 8). In Figure 7, the variation of total work-per-cycle with oscillation cycle is shown. It shows that the FD method with four updates per cycle is the fastest to converge with single update per cycle being the slowest.

The three analyses eventually converge within 0.1 per cent of each other, indicating that once convergence is achieved the results are same. This is further verified by comparing the unsteady pressure difference variation along the chord at mid-span. The comparison for the three analyses is shown in Figure 8. A very good comparison is obtained. Based on total computational cost for the analysis (Table I), in all subsequent work four updates per oscillation cycle was used for the FD method.

For the subsonic inflow condition, the analysis was also carried out using four blade-passages to simulate the −90° IBPA condition. The usage of four blade-passage analysis provides the most accurate results, as no approximations are involved. Comparisons with MP analysis using four

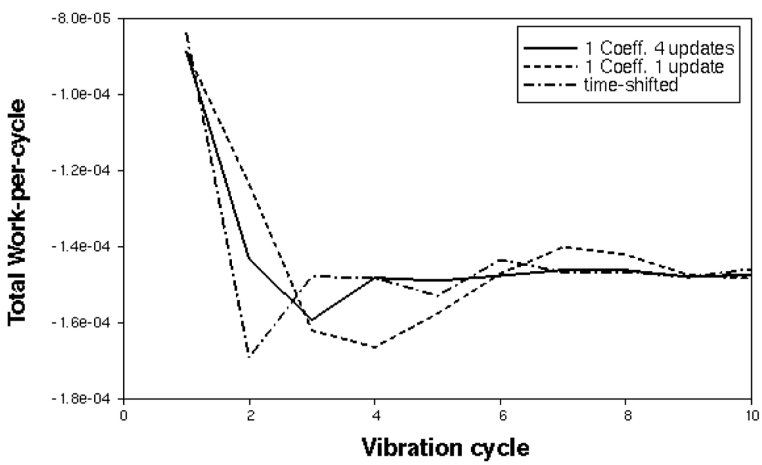


Figure 7.
Comparison of rate of convergence for phase-lagged analyses for $M_\infty = 0.7$ and $-90°$ IBPA

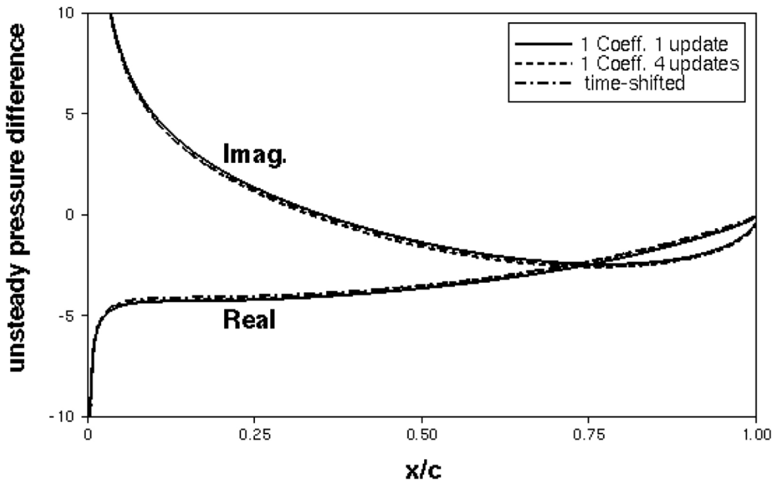


Figure 8.
Comparison of unsteady pressure difference variation with chord at mid-span for phase-lagged analyses for $M_\infty = 0.7$ and $-90°$ IBPA

Version	CPU time/time-step (s)	CPU time for convergence	CPU mem. (Mws)
Four passages in core storage	27.36	7 h 45 min	67
TS in core storage	6.94	2 h 30 min	34
TS SSDs used for I/O of BCs	6.98	2 h 35 min	15
Fourier-decomposition 1 coefficient, update	7.29	4 h	15
Fourier-decomposition 1 coefficient, updates	7.67	2 h 15 min	16

Table I.
Comparison of computer resources required by various methods for $M_\infty = 0.7$ and $-90°$ IBPA

blade-passages provide a means to measure accuracy and the benefits of phase-lagged boundary conditions in terms of savings of computer resources. The results obtained for the four-passage analysis are compared with that obtained from the TS and FD methods. The convergence of work-per-cycle for the three analysis methods is shown in Figure 9. For the four-passage analysis, as expected, the total work for all the four passages coalesce. Also, the MP takes approximately four to five oscillation cycles to converge. The TS analysis, shown with dashed lines, requires approximately eight cycles, whereas the FD method converges in five to six oscillation cycles. The total work from the three analysis methods converge to within 0.5 per cent of each other. This indicates that the three methods are equally accurate. This is further confirmed by comparing the unsteady pressure difference variation (Figure 10). Once again a very good comparison is obtained.

From these results it can be seen that the MP analysis requires the least number of oscillation cycles to converge, whereas the TS analysis requires the

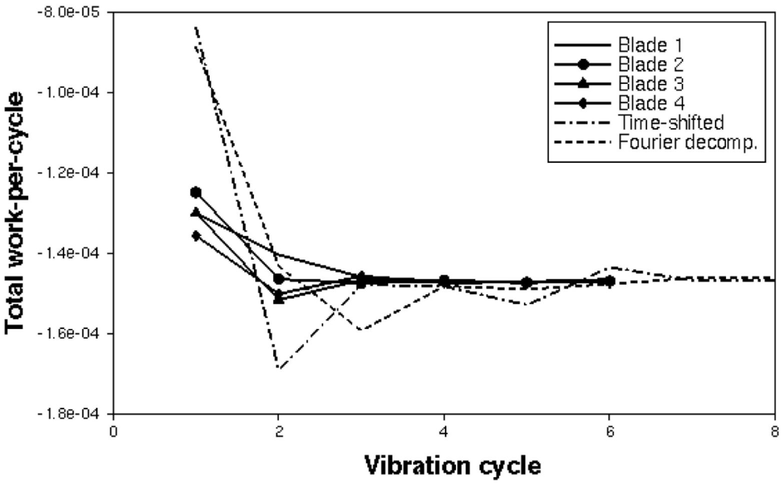


Figure 9.
Comparison of work-per-cycle convergence for phase-lagged analyses with multiple passage analysis for $M_\infty = 0.7$ and $-90°$ IBPA

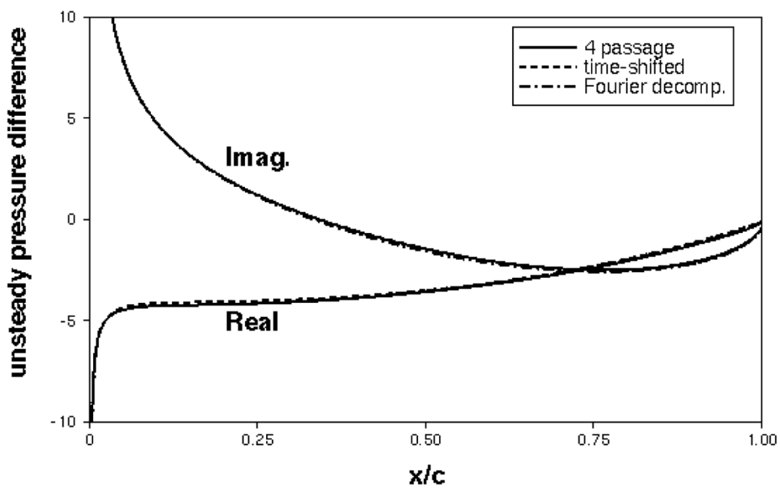


Figure 10.
Comparison of unsteady pressure difference variation with chord at mid-span of phase-lagged analyses with multiple passage analysis for $M_\infty = 0.7$ and $-90°$ IBPA

largest number of cycles. However, the computational cost for the MP analysis was largest. This is because the analysis had to be carried out using four-passages as opposed to a single passage for the two phase-lagged analyses. This reduced the problem size of the phase-lagged analysis to one-fourth that of the MP analysis. The smaller number of cycles required for convergence do not sufficiently offset the increase in computational cost for the MP analysis. Further, despite the difference in rate of convergence for the TS analysis and the FD method, the computational costs required for convergence are fairly comparable (Table I). This is due to the increased computational cost required by the FD method to Fourier decompose and regenerate the fluid properties and is offset by increased rate of convergence. It should also be noted that using the solid state devices (SSDs) could reduce the memory requirements of the TS analysis. The SSDs help to reduce the memory requirements significantly for a marginal increase in the computation cost of reading and writing to the disk.

All the above analyses were carried out on a CRAY C-90 computer. The computer resources required for the work-per-cycle method using the three analyses are tabulated in Table I. The CPU time required per time-step shows the additional cost per time-step for the FD method over the TS method. This increase in time, however, is more than compensated for by increased rate of convergence. Also, for the current problem, using the FD method reduces the memory by over 50 per cent as compared to the TS analysis. Also, it can be seen that despite the faster rate of convergence for the MP analysis, the CPU requirements are almost three times as much as that of the phase-lagged methods. This difference will increase significantly for analyzing the smaller IBPAs requiring many more blade passages for the MP analysis. Especially, since the rate of convergence for the TS analysis was found to be independent of the IBPA being analyzed (Srivastava *et al.*, 1998). Table I also shows that the FD method with multiple updates of the coefficient and the TS method may be comparable in CPU requirements. Therefore, the choice of a particular method will depend on the available resources and the problem being analyzed.

The two methods were next applied to a supersonic inflow condition to help evaluate the effectiveness and problems that FD method might have for flows with shocks. The analysis was carried out for +90° IBPA for a relative inflow Mach number of 1.3. Results are presented for analyses with one, five, and 11 Fourier coefficients. The analysis was also carried out using 15 coefficients, but the results were found to be identical to the 11-coefficient analysis.

The comparison of the total work convergence history for the three FD analyses and the TS analysis is shown in Figure 11. For sake of computational cost, the analysis was carried out for only 18 oscillation cycles. Even though the flowfield is not completely converged after 18 cycles, the disturbances appear to be dying out. The 11 coefficient analysis compares very well with the TS analysis. The one and five coefficient analyses show small differences. Interestingly, the total work calculated using only one coefficient compares very favorably with TS analysis and also converges faster. However, significant differences are found for the unsteady pressures between the results from one coefficient analysis and the other three analyses. The first and second harmonics of the blade surface unsteady pressure differences at mid-span are shown in Figure 12. The first harmonic pressure shows good comparison between the four analyses over most of the blade chord. Over the last 10-12 per cent of the chord, one coefficient analysis shows differences in both real and imaginary pressures. The second harmonic of the pressure difference, on the other hand, shows significant differences between the one coefficient Fourier analysis and the other three analyses. The TS, five- and 11 coefficient analyses

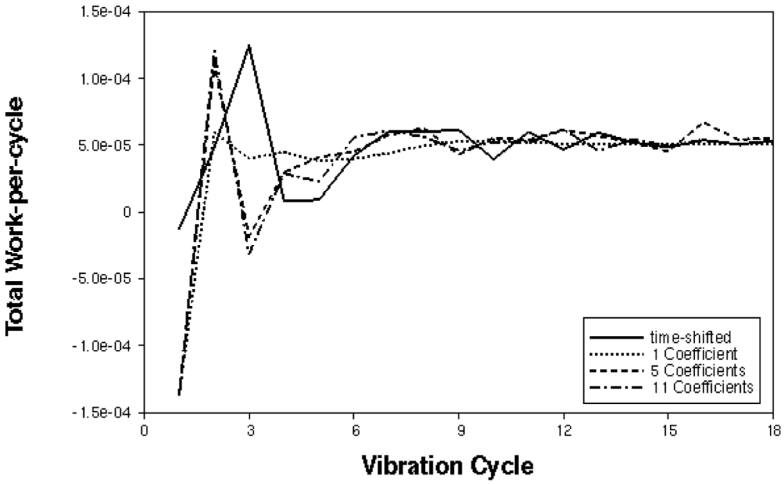
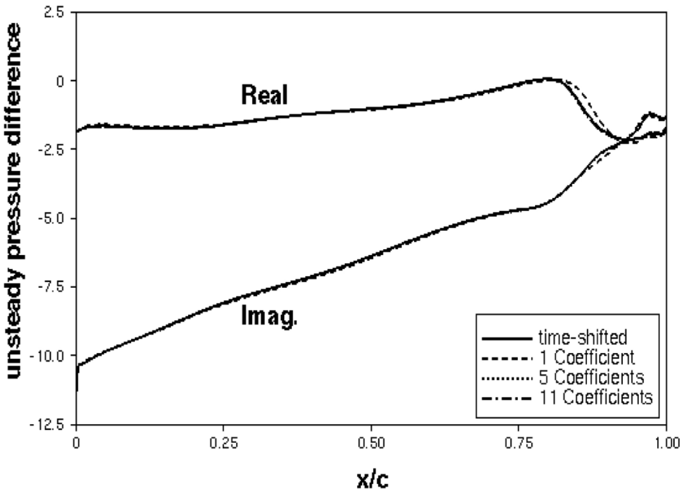


Figure 11.
Comparison of work-per-cycle history for $M_\infty = 1.3$ and +90° IBPA

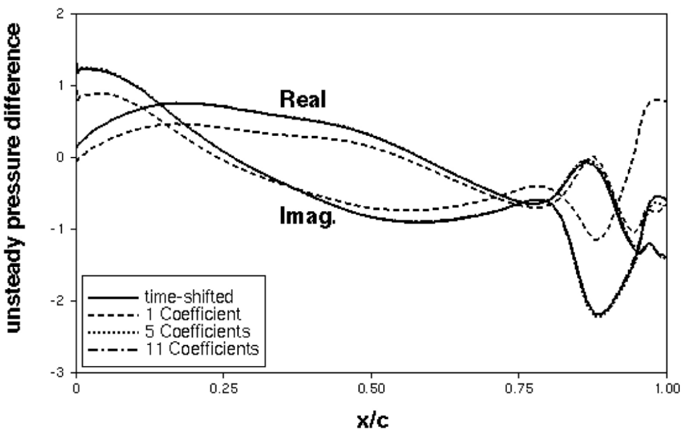
show good comparison with each other over most of the chord, with some minor differences near the trailing edge for the five-coefficient analysis. The 11 coefficient analysis compares very well with the TS method. Flow details from both analysis methods showed very good comparison with each other provided sufficient Fourier coefficients were included in the analysis for the supersonic flows.

The analysis was next applied to a turbine configuration. A linear cascade of turbine blades with large flow turning (112°) was tested at Rolls Royce Allison for several expansion ratios, exit Mach numbers and IBPAs (Rothrock *et al.*, 1981). The expansion ratio of 2.713, exit Mach number of 1.23 undergoing a pitching motion at a frequency of 340 Hz and 180° IBPA (roughly 0.5 reduced frequency based on chord) was chosen for the present study. The linear cascade was modeled as a stator with very high hub to tip ratio for the TURBO-AE analysis. A grid size of $129 \times 9 \times 33$ was used to model the blade passage. Once again, an inviscid calculation was carried out. Results for this configuration are discussed in detail in Part II of the paper. Only the unsteady results for one configuration are shown here to highlight the comparisons of the two phase-lagged methods.

The amplitude and phase of the unsteady pressure variation are shown in Figures 13 and 14. A good agreement of the unsteady pressure is observed with data for both amplitude and phase over most of the chord except near the trailing edge. In experiments, a transducer located near 30 per cent of chord on the pressure surface was not operational, hence the accuracy of the prediction in that region cannot be assessed. Also, the two phase-lagged methods compare very well with each other. Including more Fourier coefficients in the analysis can minimize the minor differences between the two methods. Comparison of



(a) First Harmonic



(b) Second Harmonic

Figure 12.
Comparison of unsteady pressure difference variation with chord for $M_\infty = 1.3$ and $+90°$ IBPA

pressure contour plots also did not show any significant difference between the two methods.

These results indicate that both TS and FD methods provide the same level of accuracy. For the two supersonic flow conditions analyzed here, the FD method results were identical to the TS method, provided sufficient Fourier coefficients were retained in the boundary condition application. However, a significant reduction in memory requirement was possible for the FD method over the TS method. Although for supersonic flows, this benefit was offset somewhat, because more Fourier coefficients were needed. It should also be

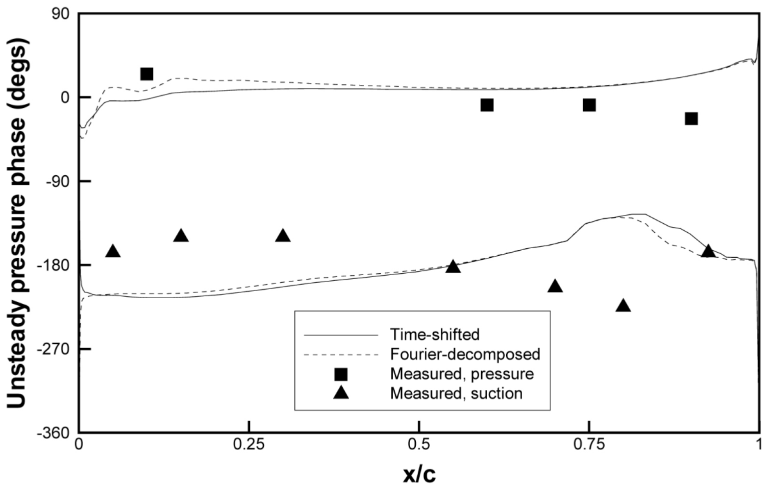


Figure 13.
Unsteady pressure amplitude comparison for the turbine cascade

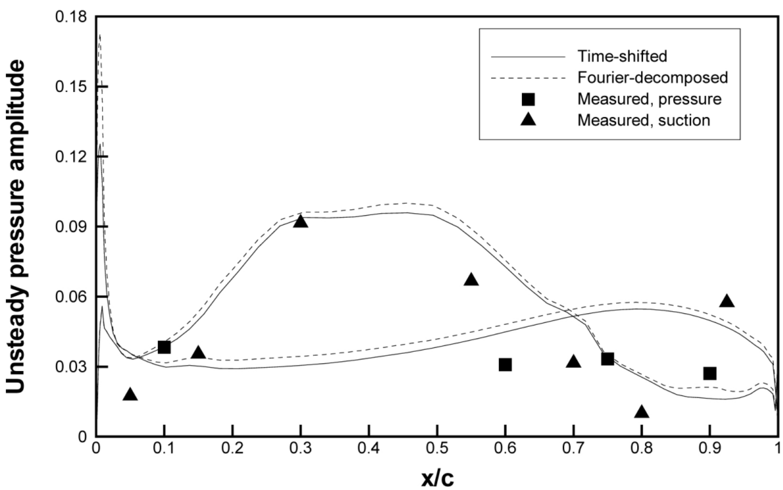


Figure 14.
Unsteady pressure phase comparison for the turbine cascade

noted that using the SSDs, the memory requirements of the TS method can be made comparable to the FD methods without significant penalty in CPU time.

Conclusions

Two phase-lagged boundary condition methods have been successfully implemented into the TURBO-AE, an Euler/Navier-Stokes based aeroelastic solver. Both these methods along with the MP analysis method have been applied to an identical geometry to investigate the accuracy and efficiency of the various methods. Comparing the results obtained from these methods it was found that all methods provide equally accurate results. The errors introduced in reconstructing the flowfield at the passage boundary using the Fourier coefficients were negligible if large number of Fourier coefficients were included in the analysis.

The study also showed that the phase-lagged methods significantly reduce the computational cost as compared to the MP analysis. These savings will be much more significant for smaller IBPAs. The CPU cost for both phase-lagged methods was found to be comparable. Because of the large reduction in memory requirements, it is recommended that FD method be used. However, depending upon the nature of the flowfield, care must be taken to include sufficient Fourier coefficients in the analysis.

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